Lecture 8

9. INDUCTIVE-COUPLED CIRCUITS

9.1. Circuits with Magnetic Coupling

Electric circuits in which processes interact by means of the common electric or magnetic field are called coupled. Circuits with a common magnetic field are referred to as magnetically or inductively coupled circuits. Consider magnetic fluxes and magnetic-flux linkages in circuits with magnetic coupling. In Fig. 9.1 two inductively coupled coils W_1 and W_2 are presented schematically (W_1, W_2 are the number of turns of the first and second coils). Here the current i_1 in the coil W_1 creates the self-induction flux of the first coil

$$\Phi_{11} = \Phi_{s1} + \Phi_{21},$$

where Φ_{s1} — the flux-leakage of the first coil (part of the flux Φ_{11} crossing the turns of the first coil only); Φ_{21} — the mutual induction flux of the second coil (part of the flux Φ_{11} crossing the turns of the second coil).

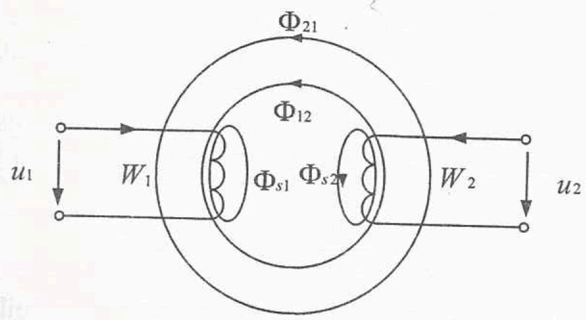


Fig. 9.1

Similarly, the current i_2 in the coil W_2 creates the self-induction flux of the second coil

$$\Phi_{22} = \Phi_{s2} + \Phi_{12},$$

where Φ_{s2} — the flux-leakage of the second coil (part of the flux Φ_{22} crossing the turns of the second coil only); Φ_{12} — the mutual induction flux of the first coil (part of the flux Φ_{22} crossing the turns of the first coil).

The full magnetic fluxes of the coils

$$\Phi_1 = \Phi_{s1} + \Phi_{21} \pm \Phi_{12} = \Phi_{11} \pm \Phi_{12},$$

$$\Phi_2 = \Phi_{s2} + \Phi_{12} \pm \Phi_{21} = \Phi_{22} \pm \Phi_{21}.$$

The product of the flow and the number of turns is called magnetic flux-linkage ψ. Then for the first and second coils we get

$$\psi_1 = W_1\Phi_1 = W_1\Phi_{11} \pm W_1\Phi_{12} = \psi_{11} \pm \psi_{12},$$
$$\psi_2 = W_2\Phi_2 = W_2\Phi_{21} = \psi_{22} \pm \psi_{21},$$

where ψ_{11}, ψ_{22} — self-induction flux-linkages of the first and second coils; ψ_{12}, ψ_{21} — mutual induction flux-linkage of the first coil (from the second coil) and that of the second coil (from the first coil) respectively.

The ratio of the mutual induction linkage to the self-induction of a coil is called the degree of coupling

$$K_{21} = \frac{\psi_{21}}{\psi_{11}},\ K_{12} = \frac{\psi_{12}}{\psi_{22}}. \tag{9.1}$$

Here K_{21} — degree of coupling of the second coil with the first coil (shows the flux-linkage of the second coil coupled with the first coil relative to the flux-linkage of the first coil); K_{12} — degree of coupling of the first coil with the second coil (shows the flux-linkage of the first coil coupled with the second coil relative to the flux-linkage of the second coil).

The ratio of a magnetic-flux linkage to the current this linkage is created by is called inductance L. There are self-induction inductances (of the first and the second coils)

$$L_1 = \frac{\psi_{11}}{i_1} = \frac{W_1\Phi_{11}}{i_1},\ L_2 = \frac{\psi_{22}}{i_2} = \frac{W_2\Phi_{22}}{i_2}, \tag{9.2}$$

mutual induction inductances (of the first and the second coils)

$$M_{12} = \frac{\psi_{12}}{i_2} = \frac{W_1\Phi_{12}}{i_2},\ M_{21} = \frac{\psi_{21}}{i_1} = \frac{W_2\Phi_{21}}{i_1}, \tag{9.3}$$

and leakage inductances (of the first and the second coils)

$$L_{S1} = \frac{\psi_{S1}}{i_1} = \frac{W_1\Phi_{12}}{i_1};\ L_{S2} = \frac{\psi_{S2}}{i_2} = \frac{W_2\Phi_{S2}}{i_2}.$$

Now the degrees of coupling from (9.1)–(9.3) are:

$$K_{21} = \frac{\psi_{21}}{\psi_{11}} = \frac{M_{21}i_1}{L_1 i_1} = \frac{M_{21}}{L_1};\ K_{12} = \frac{\psi_{12}}{\psi_{22}} = \frac{M_{12}i_2}{L_2 i_2} = \frac{M_{12}}{L_2}.$$

The geometric mean of the degrees of coupling K_{21} and K_{12} are called the coefficients of coupling

$$K = \sqrt{K_{21}K_{12}} = \sqrt{\frac{M_{21}M_{12}}{L_1 L_2}}.$$

For linear circuits $M_{21} = M_{12} = M$. Therefore

$$K = \frac{M}{\sqrt{L_1 L_2}}. \tag{9.4}$$

According to the electromagnetic induction law, the voltage at the terminals of an inductance coil is a time derivative of its total flux linkage:

$$U_1 = \frac{d\psi_1}{dt} = \frac{d\psi_{11}}{dt} \pm \frac{d\psi_{12}}{dt} = L_1\frac{di_1}{dt} \pm M_{12}\frac{di_2}{dt} = U_{11} \pm U_{12}; \tag{9.5}$$

$$U_2 = \frac{d\psi_2}{dt} = \frac{d\psi_{22}}{dt} \pm \frac{d\psi_{21}}{dt} = L_2\frac{di_2}{dt} \pm M_{21}\frac{di_1}{dt} = U_{22} \pm U_{21}. \tag{9.6}$$

Here $u_{11} = L_1\frac{di_1}{dt}$; $u_{22} = L_2\frac{di_2}{dt}$ — self-induction voltage of the first and second coils; $u_{12} = M_{12}\frac{di_2}{dt}$; $u_{21} = M_{21}\frac{di_1}{dt}$ — mutual induction voltage of the first and second coils.

Expressions (9.5) and (9.6) can be represented in complex form:

$$\begin{cases} \dot U_{m1} = j\omega L_1 \dot I_{m1} \pm j\omega M_{12}\dot I_{m2}; \\ \dot U_{m2} = j\omega L_2 \dot I_{m2} \pm j\omega M_{21}\dot I_{m1}. \end{cases} \tag{9.7}$$

In expressions (9.5), (9.6), (9.7) the double sign ± indicates that the flux, flux-linkage or the mutual induction voltage of a given coil are unidirectional with the flux, flux-linkage or the self-induction voltage of this coil (+) or they are oppositely directed (–). Hence, there are aiding and opposing connections of inductance coils. Aiding connection refers to connection of two coils at which their magnetic self-induction fluxes and the mutual induction fluxes coincide in direction. In an opposing connection – they are oppositely directed.

Hence we distinguish between like and unlike coil terminals. Like terminals refer to terminals of two coils the currents in which are unidirectional with respect to these terminals and create magnetic fluxes of the same direction, that is the coils are aiding. In the case of unlike terminals — the coils are opposing.

On circuit diagrams like terminals are denoted by points. The points on the windings W_1 and W_2 (Fig. 9.1) show that these are like terminals.

9.2. Series Connection of Magnetically Coupled Coils

Two coils with inductances L_1 and L_2 are connected in series (Fig. 9.2). Here r_1 and r_2 are active resistances of the coil. $M_{21} = M_{12} = M$.

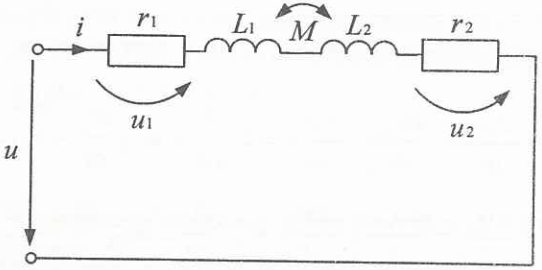


Fig. 9.2

In accordance with Kirchhoff's voltage low, we can write for this connection:

$$u = u_1 + u_2 = r_1 i + L_1\frac{di_1}{dt} \pm M_{12}\frac{di_2}{dt} + r_2 i_2 + L_2\frac{di_2}{dt} \pm M_{21}\frac{di_1}{dt}; \qquad (9.8)$$

$$u = (r_1 + r_2)i + (L_1 + L_2 \pm 2M)\frac{di}{dt} = r_e i + L_e\frac{di}{dt}, \qquad (9.9)$$

where $r_e = (r_1 + r_2)$, $L_e = (L_1 + L_2 + 2M)$ — equivalent resistance and inductance of the two series-connected magnetically coupled coils.

In complex form we get from (9.8) and (9.9):

$$\dot{U}_m = r_1\dot{I}_{m1} + j\omega L_1\dot{I}_{m1} \pm j\omega M_{12}\dot{I}_{m2} + r_2\dot{I}_{m2} + j\omega L_2\dot{I}_{m2} \pm$$
$$\pm j\omega M_{21}\dot{I}_{m1} = (r_1 + r_2)\dot{I}_m + j\omega(L_1 + L_2 \pm 2M)\dot{I}_m = \qquad (9.10)$$
$$= r_e\dot{I}_m + j\omega L_e\dot{I}_m = (r_e + jx_e)\dot{I}_m = Z_e\dot{I}_m.$$

Here

$$x_e = \omega L_e;\ Z_e = r_e + jx_e.$$

In the above expressions, the plus sign corresponds to the aiding connection of coils, the minus sign — to the opposing. We can see that for the aiding connection the equivalent inductance of two magnetically coupled coils is more by $2M$:

$$L_{ea} = L_1 + L_2 + 2M$$

whereas for the opposing connection it is less by $2M$:

$$L_{eo} = L_1 + L_2 - 2M$$

than the equivalent inductance of two coils that are not coupled magnetically:

$$L_e = L_1 + L_2.$$

This property is used in the variometer — a device designed for smooth regulation of inductance. It consists of two series-connected coils one of which is nested within the other about which it can rotate. Changing the angles of the coils axes from zero to 180 degrees, we can smoothly vary the inductance in the range from the aiding $L_1 + L_2 + 2M$ to the opposing $L_1 + L_2 - 2M$ connection of the coils, i.e. up to $4M$.

According to (9.10), vector diagrams can be constructed for the aiding and opposing connections of coils (Fig. 9.3). Here Fig. 9.3, *a* corresponds to the aiding connection. As it takes place, the self-induction voltage $j\omega L_1\dot{I}_m$ summed with the mutual induction voltage $j\omega M\dot{I}_m$ of the first coil, and the self-induction voltage $j\omega L_2\dot{I}_m$ is summed with the mutual induction voltage $j\omega M\dot{I}_m$ of the second coil. As a result, the phase angles for both coils φ_1 and φ_2 as well as the resultant phase angles of two magnetically coupled coils are positive ($\varphi_1 > 0$, $\varphi_2 > 0$, $\varphi > 0$). Each of the coils and the two coils together are generally of inductive nature. Fig. 9.3, *b* corresponds to the opposing connection.

Here the mutual induction voltage $j\omega M\dot{I}_m$ of the first coil is deducted from the self-induction voltage $j\omega L_1\dot{I}_m$ of this coil, and the mutual induction voltage $j\omega M\dot{I}_m$ of the second coil is deducted from the self-induction voltage $j\omega L_2\dot{I}_m$ of this coil. However, as for both coils the following inequalities hold:

$$j\omega M\dot{I}_m < j\omega L_1\dot{I}_m;\ j\omega M\dot{I}_m < j\omega L_2\dot{I}_m,$$

the phase angles remain $\varphi_1 > 0$, $\varphi_2 > 0$, $\varphi > 0$. That is, each coil separately and the two coils together are generally of inductive nature.

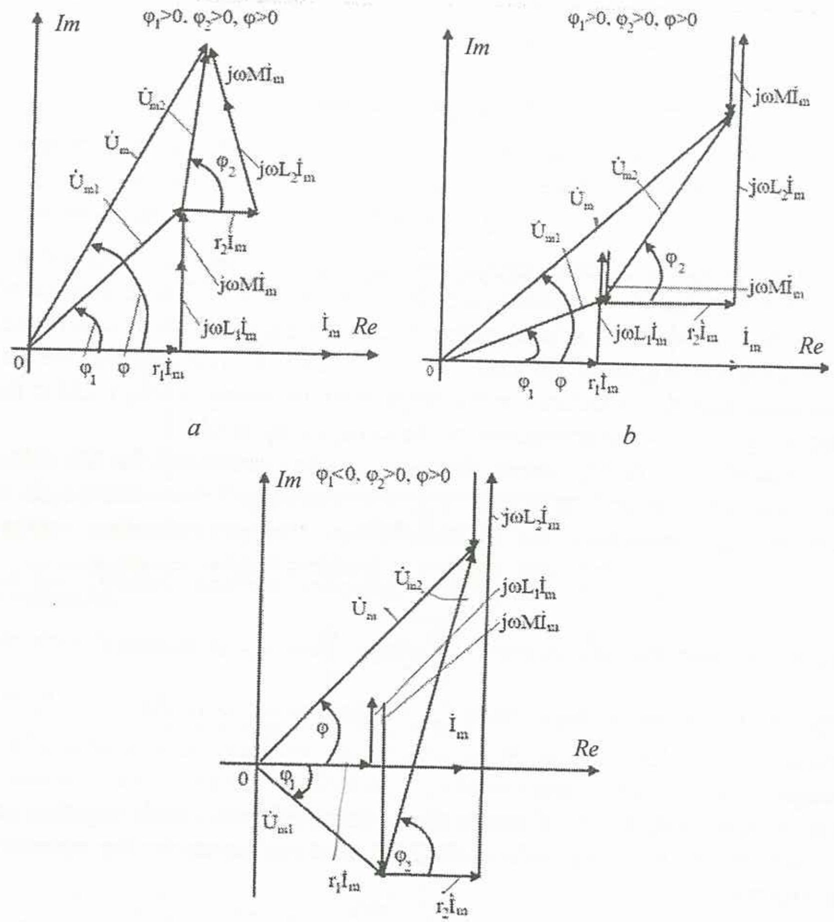


Fig. 9.3

The reactance of the second coil remains positive, the angle $\varphi_2 > 0$, and the second coil is of inductive nature.

Nevertheless, the resultant inductance of both coils is of inductive nature.

The effect that one of the coils is of capacitive nature takes place when the inductance L_2 of the second coil is significantly greater than the inductance L_1 of the first coil, which is possible with $W_2 > W_1$.

9.3. Parallel Connection of Magnetically Coupled Coils

Two coils with inductances L_1 and L_2 are connected in parallel (Fig. 9.4). Here r_1, r_2 are the resistance of these coils. Here also $M_{21} = M_{12} = M$.

In accordance with Kirchhoff's voltage law, for this connection we can write in complex form:

$$\begin{cases} r_1\dot I_{m1} + j\omega L_1\dot I_{m1} \pm j\omega M\dot I_{m2} = (r_1 + j\omega L_1)\dot I_{m1} \pm j\omega M\dot I_{m2} = \dot U_m; \\ r_2\dot I_{m2} + j\omega L_2\dot I_{m2} \pm j\omega M\dot I_{m1} = (r_2 + j\omega L_2)\dot I_{m2} \pm j\omega M\dot I_{m1} = \dot U_m \end{cases}$$

or for $M_{21} = M_{12} = M$:

$$\begin{cases} r_1\dot I_{m1} + j\omega L_1\dot I_{m1} \pm j\omega M\dot I_{m2} = \\ = (r_1 + j\omega L_1)\dot I_{m1} \pm j\omega M\dot I_{m2} = \dot U_m; \\ r_2\dot I_{m2} + j\omega L_2\dot I_{m2} \pm j\omega M\dot I_{m1} = \\ = (r_2 + j\omega L_2)\dot I_{m2} \pm j\omega M\dot I_{m1} = \dot U_m. \end{cases} \quad (9.11)$$

Write the system (9.11) in matrix form:

$$\begin{bmatrix} Z_1 \pm Z_m \\ \pm Z_m Z_2 \end{bmatrix}\begin{bmatrix} \dot I_{m1} \\ \dot I_{m2} \end{bmatrix} = \begin{bmatrix} \dot U_m \\ \dot U_m \end{bmatrix},$$

where

$$Z_1 = r_1 + j\omega L_1,\ Z_2 = r_2 + j\omega L_2,$$
$$Z_m = j\omega M.$$

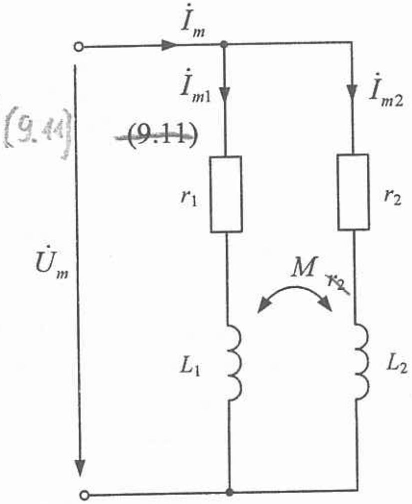


Fig. 9.4

Hence

$$\dot I_{m1} = \frac{\Delta_1}{\Delta};\quad \dot I_{m2} = \frac{\Delta_2}{\Delta}, \quad (9.12)$$

where

$$\Delta = \begin{bmatrix} Z_1 \pm Z_M \\ \pm Z_M Z_2 \end{bmatrix};\ \Delta_1 = \begin{bmatrix} \dot U_m \pm Z_M \\ \dot U_m Z_2 \end{bmatrix};\ \Delta_2 = \begin{bmatrix} Z_1\dot U_m \\ \pm Z_M\dot U_m \end{bmatrix}. \quad (9.13)$$

Now from (9.12) and (9.13) we have

$$\dot I_{m1} = \frac{\dot U_m(Z_2 \pm Z_M)}{Z_1Z_2 - Z_M^2},$$

$$\dot{I}_{m2} = \frac{\dot{U}_m(Z_1 \pm Z_M)}{Z_1 Z_2 - Z_M^2},$$

$$\dot{I}_m = \dot{I}_{m1} + \dot{I}_{m2} = \frac{\dot{U}_m(Z_1 + Z_2 \pm 2Z_M)}{Z_1 Z_2 - Z_M^2}.$$

For $r_1 = r_2 = 0$ we get:

$$\dot{I}_m = \frac{\dot{U}(j\omega L_1 + j\omega L_2 \pm 2j\omega M)}{j\omega L_1 j\omega L_2 - (j\omega M)^2} =$$

$$= \frac{\dot{U}_m}{j\omega \dfrac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}} = \frac{\dot{U}_m}{j\omega L_e}. \qquad (9.14)$$

Here

$L_e = \dfrac{L_1 L_2 - M^2}{L_1 + L_2 \pm 2M}$ — equivalent inductance of two parallel-connected coils.

In (9.14) the upper sign in the denominator (*minus*) corresponds to the aiding connection and the lower sign (*plus*) — to the opposing connection of the coils. From this we see that for the aiding connection the equivalent inductance is

$$L_{ea} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M}$$

more, and for the opposing connection it is

$$L_{eo} = \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M}$$

less than the equivalent inductance of two parallel coils that are not connected inductively ($M = 0$):

$$L_e = \frac{L_1 L_2}{L_1 + L_2}.$$

The variometer, referred to in 9.2, can also be built if magnetically coupled coils are connected in parallel. Changing the angle between the

axes of the coils from zero to 180 degrees, the inductance varies in the range from L_{ea} to L_{eo}, that is up to the value

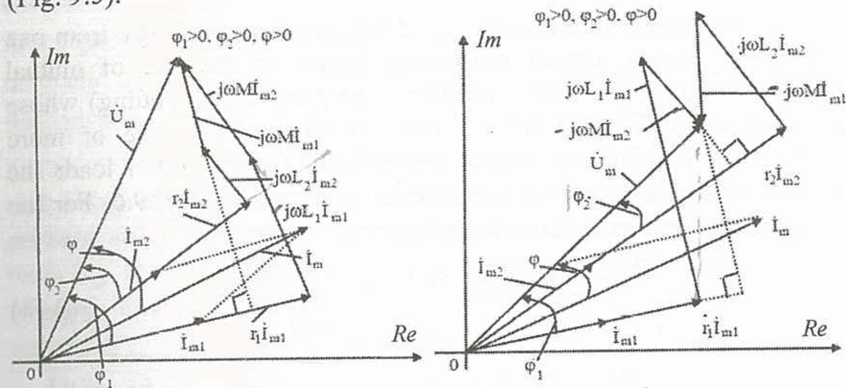
$$L_{ea} - L_{eo} = \frac{L_1 L_2 - M^2}{L_1 + L_2 - 2M} - \frac{L_1 L_2 - M^2}{L_1 + L_2 + 2M} = \frac{4M(L_1 L_2 - M^2)}{(L_1 + L_2)^2 - 4M^2}.$$

With $L_1 = L_2$ we get

$$L_{ea} - L_{eo} = \frac{4M(L_1^2 - M^2)}{4L_1^2 - 4M^2}$$

that is, up to M. It follows from this that for building a variometer it is more expedient to use a series connection of magnetically coupled coils.

In accordance with (9.11) let us construct vector diagrams for both aiding and opposing connections of magnetically coupled coils (Fig. 9.5).



a b

Fig. 9.5

Here Fig. 9.5, *a* corresponds to the aiding connection. In this case the self-induction voltage of the first coil $j\omega L_1 \dot{I}_{m1}$ is summed geometrically with the mutual induction voltage $j\omega M \dot{I}_{m2}$ of the coil, and the self-induction voltage of the second coil $j\omega L_2 \dot{I}_{m2}$ is summed geometrically with the mutual induction voltage $j\omega M \dot{I}_{m1}$ of the coil, the result being the voltage $\dot{U}_m$ applied to the coils. The phase angles of the coils φ_1 and φ_2 as well as the resultant phase angle φ are positive ($\varphi_1 > 0$, $\varphi_2 > 0$, $\varphi > 0$). Each of the coils and the two coils together are generally of inductive nature.

Fig. 9.5, b corresponds to the opposing connection. In this case, the mutual induction voltage of the first coil $j\omega L_2 \dot{I}_{m2}$ and that of the second coil $j\omega M \dot{I}_{m1}$ are oppositely directed compared to Fig. 9.5, a, and they are also summed geometrically with the self-induction voltage of the first coil $j\omega L_1 \dot{I}_{m1}$ and with that of the second coil $j\omega L_2 \dot{I}_{m2}$ respectively. However, in contrast to the series connection of magnetically coupled coils, a capacitive effect cannot be attained in one of the coils if they are connected in parallel. Always each coil separately and the two coils together are generally of inductive nature ($\varphi_1 > 0$, $\varphi_2 > 0$, $\varphi > 0$).

9.4. Ideal and Real Transformers

A transformer is a device designed for transferring energy from one part of an electric circuit to another based on the use of mutual induction. The transformer consists of a primary coil (winding) whose terminals are connected to a source of energy, and one or more secondary coils (windings) whose terminals are connected to a load. The network of a double-wound transformer is shown in Fig. 9.6. For the loops I and II we can write in complex form:

$$\begin{cases} r_1 \dot{I}_{m1} + j\omega L_1 \dot{I}_{m1} - j\omega M_{12} \dot{I}_{m2} - \dot{U}_{m1} = 0; \\ r_2 \dot{I}_{m2} + j\omega L_2 \dot{I}_{m2} - j\omega M_{21} \dot{I}_{m1} + \dot{U}_{m2} = 0. \end{cases} \quad (9.14)$$

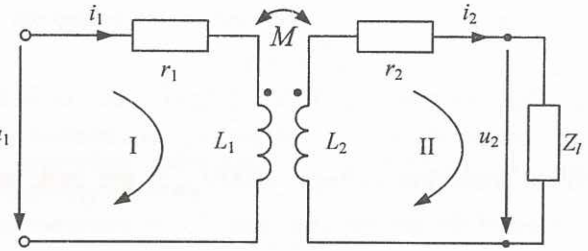


Fig. 9.6

Here, the mutual induction voltages of the first $j\omega M_{12} \dot{I}_{m2}$, and second $j\omega M_{21} \dot{I}_{m1}$ windings are taken with a minus sign, because the

winding are opposing (the currents i_1 and i_2 differently oriented relative to the same name terminals marked with dots).

Considering the transformer without a core, that is as a linear element, we consider, as noted above, $M_{12} = M_{21} = M$. Then

$$\begin{cases} (r_1 + j\omega L_1)\dot{I}_{m1} - j\omega M \dot{I}_{m2} = \dot{U}_{m1}; \\ -j\omega M \dot{I}_{m1} + (r_2 + j\omega L_2)\dot{I}_{m2} = -\dot{U}_{m2}. \end{cases} \quad (9.15)$$

From (9.15) it is obvious that the current $\dot{I}_{m2}$ through the secondary winding is directly connected with the current $\dot{I}_{m1}$ through the primary winding. Thus, if the current $\dot{I}_{m2}$ through the secondary winding increases, while the voltage $\dot{U}_{m1}$ of the primary winding is constant, the current $\dot{I}_{m1}$ through the primary winding also increases as seen from the first equation in (9.15). The physical explanation of this fact is that the magnetic flux of mutual induction Φ_{12}, created by the current $\dot{I}_{m2}$ through the secondary winding, is opposing to the magnetic flux of self-induction Φ_{11}, created by the current $\dot{I}_{m1}$ through the primary winding, and, being deducted from Φ_{11}, it reduces the total magnetic flux of the primary winding. It results in reducing the full flux linkage ψ_1 and in reducing the equivalent inductance from the side of the primary winding. Therefore, its inductive reactance decreases, and at a constant voltage $\dot{U}_{m1}$ the current $\dot{I}_{m1}$ increases.

Using (9.15) we can build an equivalent circuit of a transformer without magnetic coupling of coils (Fig. 9.7). This network is used in the analysis and calculation of transformers.

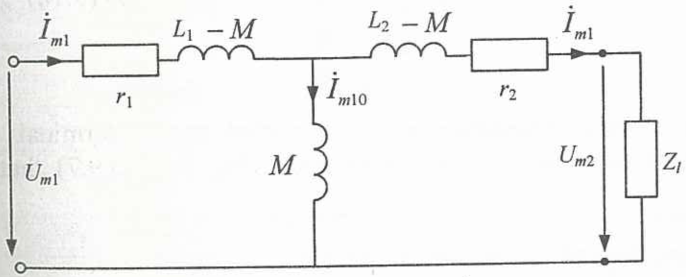


Fig. 9.7

Consider the no-load operation of a transformer: $\dot{I}_{m2} = 0$; $Z_i \to \infty$, (Fig. 9.6, 9.7). We get from (9.15):

$$\dot{I}_{m1} = \frac{\dot{U}_{m1}}{r_1 + j\omega L_1} = \dot{I}_{m10}.$$

The current $\dot{I}_{m10}$ is called no-load current or magnetization current (Fig. 9.7). Under a load this current decreases, and it tends to zero as the inductance of the primary coil increases ($L_1 \to \infty$).

From (9.15) we get for no-load operation:

$$\frac{\dot{U}_{m2}}{\dot{U}_{m1}} = \frac{j\omega M}{r_1 + j\omega L_1}. \quad (9.16)$$

For transformer analysis, it is convenient to introduce the concept of the ideal transformer, i.e. a transformer with no active losses ($r_1 = r_2 = 0$) and whose coefficient of coupling (9.4) is equal to unit:

$$K = \frac{M}{\sqrt{L_1 L_2}} = 1. \quad (9.17)$$

Then, from (9.16)

$$\frac{\dot{U}_{m1}}{\dot{U}_{m2}} = \frac{L_1}{M} = \sqrt{\frac{L_1}{L_2}} = n. \quad (9.18)$$

The value n is called the transformation coefficient. As it follows from (9.2) and (9.3):

$$L_1 = \frac{W_1 \Phi_{11}}{i_1} = \frac{W_1(\Phi_{21} + \Phi_{S1})}{i_1}; \quad M = \frac{W_2 \Phi_{21}}{i_1}$$

and $\Phi_{s1} = \Phi_{s2} = 0$, which follows from (9.17), then from (9.18)

$$n = \frac{W_1}{W_2}.$$

Let a load impedance Z_i is connected to the terminals of the secondary winding of an ideal transformer (Fig. 9.6 and 9.7), that is

$$\dot{U}_{m2} = \dot{I}_{m2} Z_i. \quad (9.19)$$

Then, from (9.15) and (9.19) we get

$$\begin{bmatrix} j\omega L_1 & -j\omega M \\ j\omega M & -(j\omega L_2 + Z_i) \end{bmatrix} \begin{bmatrix} \dot{I}_{m1} \\ \dot{I}_{m2} \end{bmatrix} = \begin{bmatrix} \dot{U}_{m1} \\ 0 \end{bmatrix}.$$

Hence,

$$\dot{I}_{m1} = \frac{\Delta_1}{\Delta} - \frac{-\dot{U}_{m1}(j\omega L_2 + Z_i)}{-j\omega L_1(j\omega L_2 + Z_i) + (j\omega M)^2},$$

$$\dot{I}_{m2} = \frac{\Delta_2}{\Delta} - \frac{-j\omega M \dot{U}_{m1}}{-j\omega L_1(j\omega L_2 + Z_i) + (j\omega M)^2}.$$

Consider the ratio:

$$\frac{\dot{I}_{m2}}{\dot{I}_{m1}} = \frac{j\omega M}{j\omega L_2 + Z_i}$$

or with account of (9.17)

$$\frac{\dot{I}_{m2}}{\dot{I}_{m1}} = \frac{j\omega\sqrt{L_1 L_2}}{j\omega L_2 + Z_i}.$$

Usually $Z_i \ll j\omega L_2$. Then, with account of (9.18), we obtain

$$\frac{\dot{I}_{m2}}{\dot{I}_{m1}} = \sqrt{\frac{L_1}{L_2}} = n.$$

Using (9.15) we can build a vector diagram of the transformer under a load (Fig. 9.8). First, with an inductive load Z_i ($\varphi_2 > 0$) we lay off the vector of the load voltage $\dot{U}_{m2}$, then the vectors of the voltage drop across the resistance r_2 (in-phase with the current $\dot{I}_{m2}$) and across the inductance L_2 (at an angle of $+\pi/2$ to the current $\dot{I}_{m2}$). Since the sum of the voltages in the loop of the secondary winding is equal to zero according to (9.15), then, connecting the origin of coordinates with the end of the vector $j\omega L_2 \dot{I}_{m2}$, we obtain the vector of mutual induction voltage $j\omega M \dot{I}_{m1}$, which lags the current $\dot{I}_{m1}$ by an angle of $\pi/2$. Hence, we construct the vector of the primary current $\dot{I}_{m1}$, the vectors of the voltage drop across the resistance r_1 and the inductance L_1. The vector of the mutual induction voltage $j\omega M \dot{I}_{m1}$ is built at an angle of "$-\pi/2$" to the vector of the secondary current $\dot{I}_{m2}$. Connecting the the origin of coordinates with the end of the vector $j\omega M \dot{I}_{m2}$, we obtain the vector of the primary voltage $\dot{U}_{m1}$.

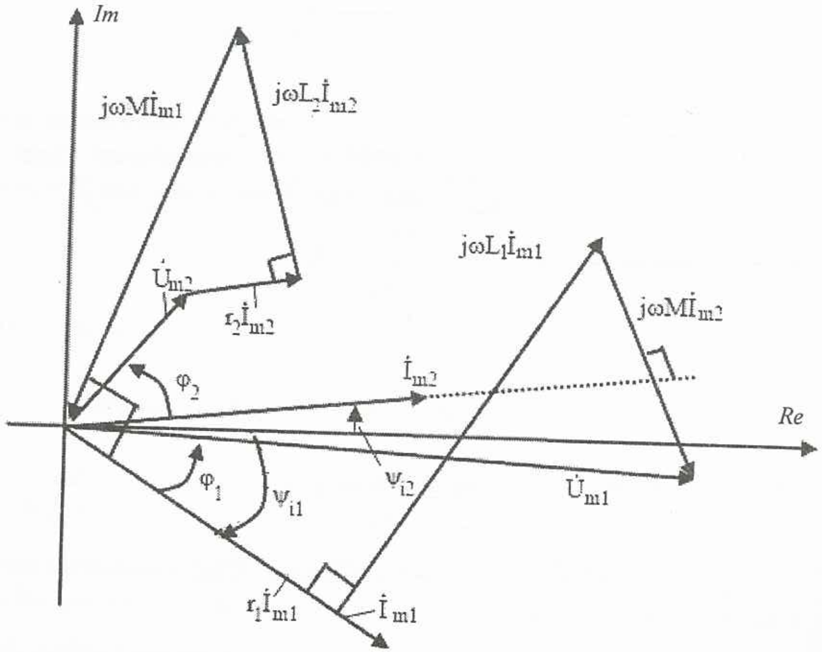


Fig. 9.8

Consider the input resistance of the primary winding of the transformer

$$Z_{in} = \frac{\dot{U}_{m1}}{\dot{I}_{m1}} = \frac{n\dot{U}_{m2}}{\frac{\dot{I}_{m2}}{n}} = n^2 Z_i.$$

That is the ideal transformer changes the load impedance n-fold without changing the argument of the impedance. This property is used for matching the load with the internal resistance of the power supply on the primary side of the transformer.

Example 1

Set up a system of equations of the electrical balance of the circuits whose diagrams are shown in Fig. 9.9, *a*, *b*.

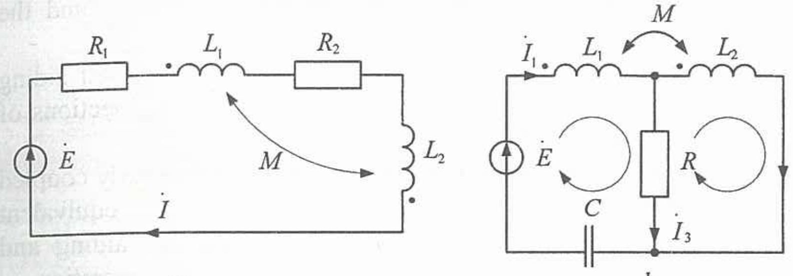


Fig. 9.9

Solution

Through the inductances L_1 and L_2 flows the same current (Fig. 9.9, *a*). *M* — mutual inductance factor. So, the mutual inductance voltages across the inductances L_1 and L_2 are the same. The equation has the form

$$-\dot{E} + R_1\dot{I} + j\omega L_1\dot{I} - j\omega M\dot{I} + R_2\dot{I} + j\omega L_2\dot{I} - j\omega M\dot{I} = 0.$$

For Fig. 9.9, *b* the currents flowing through the inductances L_1, L_2 and the voltages across the inductances L_1 and L_2, with the mutual inductance factor being *M*, are different. The system of equations has the form

$$\dot{I}_1 - \dot{I}_2 - \dot{I}_3 = 0;$$

$$-\dot{E} + j\omega L_1\dot{I}_1 + j\omega M\dot{I}_2 + \dot{I}_3 R + \dot{I}_1\frac{1}{j\omega C} = 0;$$

$$-\dot{I}_3 R_3 + j\omega L_2\dot{I}_2 + j\omega M\dot{I}_1 = 0.$$